Similarity measures for mid-surface quality evaluation

Similarity Measures for Mid-surface Quality Evaluation

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Abstract

Mid-surface models are widely used in engineering analysis to simplify the analysis of thin walled parts, but it can be difficult to ensure that the mid-surface model is representative of the solid part from which it was generated. This paper proposes two similarity measures that can be used to evaluate the quality of a mid-surface model by comparing it to a solid model of the same part.

Two similarity measures are proposed; firstly a geometric similarity evaluation technique based on the Hausdorff distance and secondly a topological similarity evaluation method which uses geometry graph attributes as the basis for comparison. Both measures are able to provide local and global similarity evaluation for the models.

The proposed methods have been implemented in a software demonstrator and tested on a selection of representative models. They have been found to be effective for identifying geometric and topological errors in mid-surface models and are applicable to a wide range of practical thin walled designs.

Keywords: Mid-surface; medial surface; similarity assessment.

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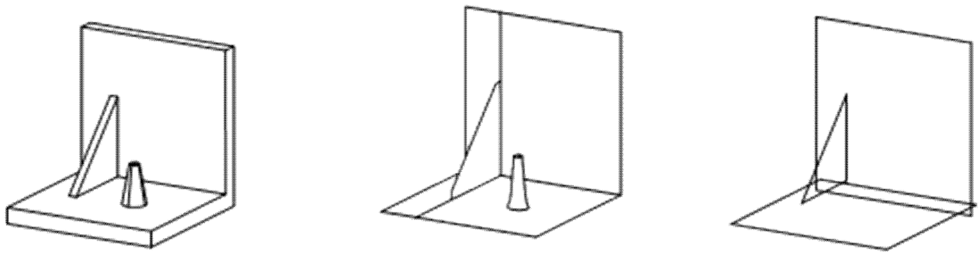
1 Introduction

Mid-surface models have received a great deal of interest in recent years because they can reduce the computational cost of performing engineering analyses on thin walled parts. The mid-surface representation of a part is a dimensionally reduced abstraction in which each wall is represented by a surface of zero thickness; for thin walled objects the mid-surfaces provide a simplified description which retains the main shape characteristics of the original design. Mid-surface models are used extensively for the analysis of thin walled parts in engineering analysis applications such as finite element analysis and mould filling/ cooling analysis [1][2][3][4]. Mid-surface models have also been used as a basis for feature recognition from thin walled moulded parts [5].

There are a number of automated mid-surface generation techniques that can create a mid-surface abstraction from a CAD solid model, but there are currently no applications that can guarantee to generate a representative mid-surface model for any arbitrary solid part. This is partly due to limitations in the current algorithms, but also because there are many part shapes for which it is not possible to generate a representative mid-surface model [6]. In practice it is often necessary for the analyst to make manual adjustments to an automatically generated mid-surface model before it can be used for engineering analysis. Figure 1. shows an example of good and poor quality mid-surface models generated using a commercial mid-surface generation tool (UGS I-DEAS NX). In Figure 1 (b) it can be seen that all of the features of the solid part are represented in the mid-surface model, whereas in Figure 1 (c) the boss has not been captured and the main walls have not been correctly connected together.



(a) **(b)** **(c)**

Figure 1. Example of Good and Poor Quality mid-surface Generation (a) Solid Model (b) Accurately Generated Mid-surface Model (c) Poor Quality Mid-surface Model

The motivation for the research presented in this paper is to assist users of mid-surface models who need to judge how well a mid-surface model represents the solid shape from which was been generated. It should be emphasised that mid-surface representations can only accurately represent parts where the wall thickness is small compared to the other dimensions, and the methods presented in this paper are oriented towards practical applications of mid-surface models. For example in finite element analysis 2D meshes constructed on the mid-surface geometry are appropriate when "the length in one of the spatial dimensions, for example the material thickness, is much less than the lengths in the other two dimensions" [7]. Similarly parts designed for injection moulding must have a thin and relatively uniform wall thickness. Bralla [8] states that "uniform wall sections will help to produce warp-free and strain-free molded parts… Dimensional variations…are accentuated by uneven or abrupt wall-thickness changes".

The objective of the research is to develop techniques that can be used to evaluate the accuracy of a mid-surface model by measuring its similarity to the solid model from which it was generated. Two techniques are presented, one to compare the geometric similarity of the two models and the second to compare their topological similarity. The remaining sections of the paper are structured as follows:- Section 2 provides a review of the relevant literature relating to mid-surface model generation and shape similarity measures and Section 3 presents the proposed similarity measures. In Section 4 the demonstrator is described, and in Section 5 case studies are presented. Section 6 discusses the results and conclusions are drawn.

2 Literature Review

The automatic generation of mid-surface abstractions from solid models has been a subject of research for many years and there has been significant progress in the field, however there is still no method that can generate an accurate mid-surface model for any arbitrary shape. The following sections provide a brief overview of existing mid-surface generation techniques, and a review of shape similarity measures that have been used to compare computer based geometry models.

2.1 Mid-surface Generation Techniques

There are a number of approaches to that have been developed for the automatic mid-surface generation from CAD solid models. Two of the main approaches are developments from the medial axis transform (MAT) and surface pairing techniques.

The medial surface transform is a three-dimensional extension of the MAT first proposed by Blum in the 1960s [9]. The medial surface is defined as the locus of the centre of a maximal sphere as it rolls around the object interior [10]. Algorithms to calculate the medial axis transform are reasonably mature, but the development of a robust algorithm for the medial surface transform is still the subject of research [6]. The surface pairing approach generates mid-surfaces by constructing surfaces between candidate pairs of the faces on the solid part, and then trimming/ extending the resultant faces to form a connected model. Rezayat [11] proposed a surface pairing algorithm and claimed that the surface-pairing approach has benefits over medial-surface techniques because the resultant geometry is cleaner and requires less reconstruction that those from medial axis approaches. However the surface pairing approach also has difficulties in implementation because it can be difficult to identify all of the surface pairs and to correctly connect the generated mid-surfaces.

Both of the above approaches have been implemented in commercial software tools (the Medial Object Toolkit [12] and UGS I-DEAS NX [13]) that are able to generate mid-surfaces for a range of realistic designs, but both have limitations in the range of shapes for which a mid-surface can be automatically generated.

There has been continued research into automatic mid-surface generation techniques in the last 10 years. One recent development of interest is the approach been proposed by Chong, Kumar and Lee [14] which uses solid decomposition to generate idealised mixed-dimensional models for finite element analysis. The authors use concavity/ convexity attributes to decompose the solid into regions for which medial surfaces can be generated. Their approach supports the generation of a mixed solid and mid-surface model suitable for finite element analysis. Their approach shows promise, but the authors acknowledge that there is still potential for errors on the mid-surface due to the mid-surface extension and stitching operations that are performed during the process. Ramanathan and Gurumoorthy have developed a mid-surface generation technique which uses a combination of the 2D medial axis and face pairing [15].

2.2 Shape Similarity Measures

Shape similarity measures have been used for a number of different applications in 3D CAD, particularly for identifying similar parts from a large database of CAD parts. Iyer et al [16] provide a comprehensive review of three-dimensional shape searching techniques, including similarity measures. They describe a wide range of shape similarity measures including global properties of the 3D model (using for example moments or spherical harmonics), geometric parameters (such as surface area to volume ratio, crinkliness, bounding box aspect ratio etc.), graph based techniques, histogram techniques and feature recognition based approaches. They also identify three distance metrics that can be used to measure similarity - the Minkowsky distance, Hausdorff distance and Correlation metric. The presented techniques can be broadly categorised into shape similarity measures and topology similarity measures. The global properties and histogram techniques are concerned only with geometric shape, whereas the graph based and feature recognition techniques can also evaluate topological similarity.

Rea, Corney, Clark and Taylor [17] present a histogram based similarity metric called the Surface Partitioning Spectrum (SPS) to identify similar parts from a database. The SPS characterises the geometry and topology of a shape into a single 2D graph which they claim provides a more sensitive measure of similarity than other techniques. They use a facetted representation of the part as the basis for similarity assessment and use the angles between adjacent facets to characterise the part shape and topology.

El-Mehalawi et al [18] present a local similarity evaluation technique based on exact and inexact graph matching. Iyer et al [19] have developed a graph based shape similarity measure using a skeletonised CAD model. In their approach the boundary representation solid model is converted to a voxel representation, and then converted to a one dimensional skeleton model using a thinning algorithm. A graph is then constructed from the skeletonised geometry and used for similarity evaluation. Iyer’s work uses inexact graph matching in combination with other similarity measures to identify similar CAD models from a database.

Several researchers have developed combined global and local measures of similarity. Chu [20] presents a shape similarity measure for CAD models using a combination of a graph based topological comparison and a shape histogram. Part similarity is initially ranked using a topology graph and then parts with identical topological similarity are differentiated using a shape histogram. Chu argues that a combined measure of topology and shape provides a better overall measure of shape similarity.

Hong, Lee and Kim [21] also perform a combined global and local similarity comparison. The global similarity is performed using shape histograms, and the local comparison is based on feature recognition, volume comparison and face counting. The local similarity comparison is used in their research as an additional differentiator between similar parts, but does not attempt to highlight similar and dissimilar regions.

The vast majority of similarity evaluation research is focussed on identifying similar parts from a database of models. Only one example has been found in the literature of a similarity measure being used to compare a solid model with a simplified model. Li and Liu [22] define a feature recognition evaluation approach in their paper on feature recognition for the removal of detailed features from CAD models. They use a volume-simplification-ratio to compare the volume of a simplified feature model with the volume of the solid model from which it was generated to allow them to compare the extent of the changes to the model from the detail removal process.

An extensive review of the relevant literature has identified that while there is a significant body of research in shape similarity, most research is concerned with global similarity measures for shape retrieval, or local similarity for models with graphs that can be compared directly. Existing global similarity measures are not appropriate for comparing a mid-surface with a solid model because they are not sensitive enough to differentiate between errors in mid-surface generation and the inherent differences between the two models. Existing graph based local measures rely on the two models for comparison having similar graph structures which is not true for mid-surface/ solid model comparison.

3 Mid-Surface Model Similarity Evaluation

An effective mid-surface model similarity evaluation technique must be able to measure the similarity of a mid-surface model to a solid model with respect to both geometry and topology. The geometric evaluation should be sensitive enough to determine whether all the design features of the solid model are represented on the mid-surface model and the topological evaluation must evaluate whether the connectivity between solid model faces is correctly represented on the mid-surface model. The similarity evaluation methodology developed in this research is presented in two parts:- section 3.1 provides an introduction to the similarity evaluation methods, section 3.2 describes the geometric similarity evaluation, and section 3.3 describes the topological similarity.

3.1 Overview of the Similarity Evaluation Method

The geometric and topological similarity evaluation methods presented in this paper locally compare solid and mid-surface models to identify dissimilar regions. The local results are then combined to provide a global measure of similarity.

In general for an accurately generated mid-surface model the distance from any point on the surface of the solid model to the closest point on its mid-surface will be half the local wall thickness. Figure 2. shows this relationship for a simple X-junction with four walls of thickness *t* in which it can be seen that for points p_1 and p_2 the distance to the closest point on the mid-surface model is 0.5*t*. Conversely the distance from a point on the mid-surface to the closest point on the solid model may be significantly greater than half the local wall thickness close to wall junctions. In the figure it can be seen that the distance from mid-surface model to solid model for the X-junction can be as much as 0.7*t* (for example at point p_4) due to the geometric effects at wall junctions. For other junction configurations both the solid to mid-surface and the mid-surface to solid distances may be equal to or greater than the local wall thickness.

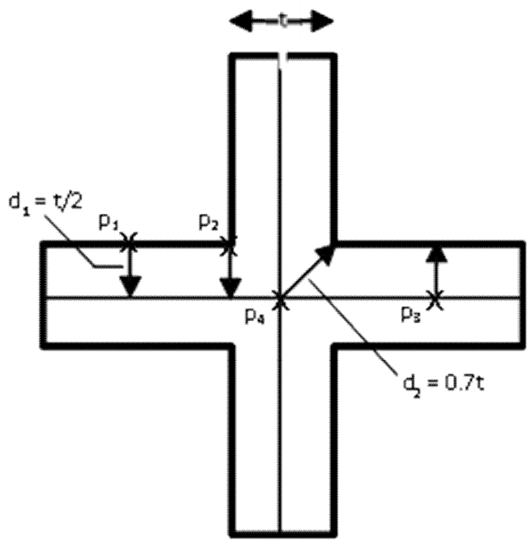


Figure 2. Solid Model to Mid-surface Distances for X-Junction

The similarity measures proposed in this research use a search radius (defined as a threshold value) to perform the similarity evaluation between the models.
The methods are therefore only applicable only to parts where the wall thickness is small relative to the other part dimensions (for practical purposes the wall thickness is assumed to be less than half the minimum of other part dimensions). It is also assumed that variations in wall thickness between regions of the part as well as across individual walls of the part are small relative to the other dimensions. This assumption is consistent with the analysis and manufacturing requirements for many types of thin walled parts.

3.2 Geometric Similarity Evaluation

The mid-surface geometric similarity evaluation proposed in this research compares the geometric shape of a mid-surface model to that of an associated solid model. A local evaluation is performed to identify dissimilar regions on the models and a global similarity index is defined to provide an overall measure of the geometric similarity between the models. The objective of the evaluation is to identify regions that are missing from, or have been incorrectly generated in the mid-surface model.

Most global measures of shape similarity such as geometric parameters and moments were found not to be suitable for this application because they are not sensitive enough to differentiate between the inherent differences between a solid model and its mid-surface and errors in mid-surface generation.

The proposed geometric similarity approach is based on the Hausdorff distance which provides a measure of the maximum dissimilarity between two similar shapes. The Hausdorff distance has been widely used in digital image processing to match similar 2D images [23] and has also more recently been applied to 3D shape matching [24][25]. An important characteristic of the Hausdorff distance for this application is that it can be applied to non-matched point sets and can provide information about both global and local similarity.

Formally the *directed Hausdorff distance* is defined as the maximum over all the points in point set *X* of the minimum distances to point set *Y*, where *d(x,y)* is the 3D distance between *x* and *y*[16]:

$$\vec{h}(X,Y) = \max_{x \in X} \min_{y \in Y} d(x,y) \qquad \textbf{(1)}$$

The *Hausdorff distance* is defined as the larger of $\vec{h}(X,Y)$ and $\vec{h}(Y,X)$.

$$H(H,Y) = \max\left\{\vec{h}(X,Y), \vec{h}(Y,X)\right\} \qquad \textbf{(2)}$$

In order to use the directed Hausdorff distance to measure the dissimilarity between a solid and mid-surface model both models must first be discretised into finite sets of points on the model surfaces. The selection of the point density is important for the accuracy of the results, and there is a trade-off between results accuracy and computation time.

If X is the set of points on the surfaces of the solid part and Y is the set of points on the mid-surface model then the directed Hausdorff distance $\vec{h}(X,Y)$ defines the maximum distance from any point on the solid model to the closest point on the mid-surface model. The distribution of minimum distances between each point on the solid model and the closest point on the mid-surface model can be displayed by plotting all the values $\min_{y \in Y} d(x,y)$. Any missing regions on the mid-surface model can be identified by groups of points where the minimum distance is greater than a threshold value related to the local wall thickness.

Similarly the reverse Hausdorff distance $\vec{h}(Y, X)$ defines the maximum distance from any point on the mid-surface model to the closest point on the solid model and can be used to identify any additional mid-surface regions that have been erroneously added to the mid-surface model.

Figure 3 shows an example of the geometric similarity results for a simple test case with missing features in the mid-surface model. The geometric similarity evaluation results can be plotted as a contour plot of minimum distances (Figure 3 (c)) or using a single threshold value (Figure 3 (d)) which makes the results easier to interpret.

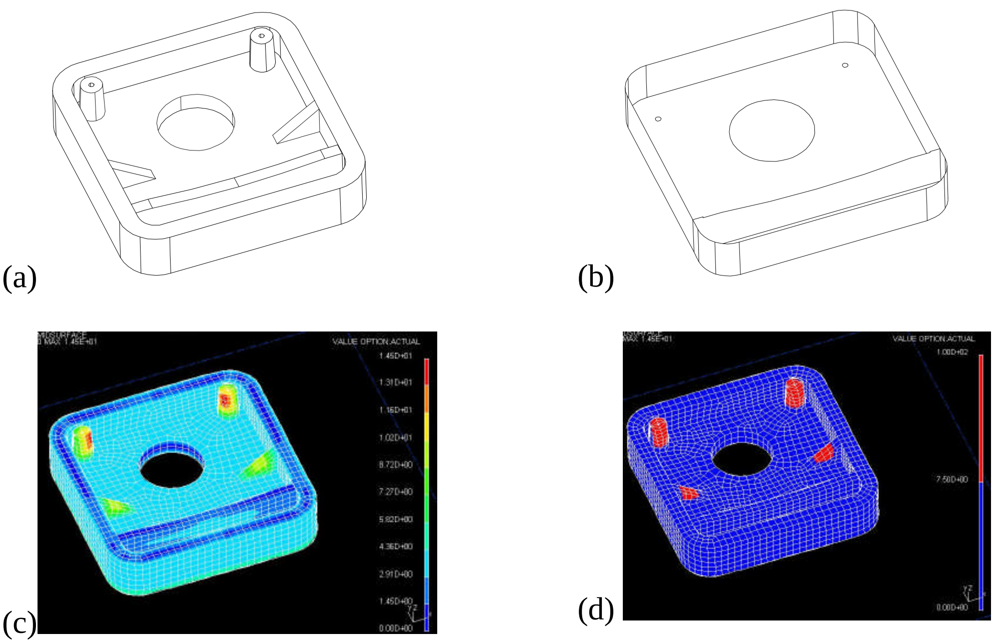


Figure 3. Geometric Similarity Results for a Simple Part

3.2.1 Geometric Similarity Index

A global measure of geometric similarity has been defined as the proportion of solid model points that are within a specified threshold distance of the mid-surface model. The global similarity index (GSI) is defined as:

$$GSI = \frac{|X_{pass}|}{|X|} \times 100 \qquad \textbf{(3)}$$

Where

$|X|$ = Number of points X on the surfaces of the solid model
$|X_{pass}|$ = Number of points in X_{pass} where $X_{pass} = \{ \min_{y \in Y} d(x, y) \mid \leq T\}$
T = Threshold value for distance evaluation

A threshold value equal to the local wall thickness has been used for testing to accommodate the geometric effects of wall junctions and the additional variability introduced by the discretising the surfaces into points for comparison. A GSI of 1 indicates a high degree of geometric similarity between two models, and a GSI of 0 indicates that there are no points within the expect threshold distance between the models.

3.3 Topological Similarity Evaluation

The topological similarity evaluation compares the topology of a mid-surface model and its associated solid model. The topology of mid-surface models is important for downstream analysis applications such as finite element analysis and feature recognition where the connectivity between mid-surface faces required. A mid-surface model with the correct geometric shape but incorrect topology may produce incorrect results in downstream analysis. The topological similarity index proposed in this research compares the convexity/ concavity characteristics of the solid model edges to the connectivity of the mid-surface edges to provide a measure of topological similarity.

3.3.1 Background

The topology of a CAD solid model can be represented using a graph structure such as the Face Adjacency Graph (FAG) [26][27] and attributes can be used to capture information about face-edge relationships, for example the concavity/ convexity of the edge. An example of a simple solid model and its FAG is shown in Figure 4 (a).

The topology of a mid-surface model cannot be represented using this form of graph because it violates that basic requirement of that graph that every edge must be connect exactly two faces. Mid-surface models can be represented using an alternative graph structure such as the Attributed Mid-surface Adjacency Graph (AMAG) proposed by the authors and described in [5]. Attributes can be associated with each edge to represent the edge "order", where the order of an edge refers to the number of adjacent faces using it. The mid-surface representation and geometry graph for the model are shown in Figure 4 (b). It can be observed from the figure that the solid and mid-surface models cannot be compared directly using their geometry graphs because their graphs have fundamentally different structures.

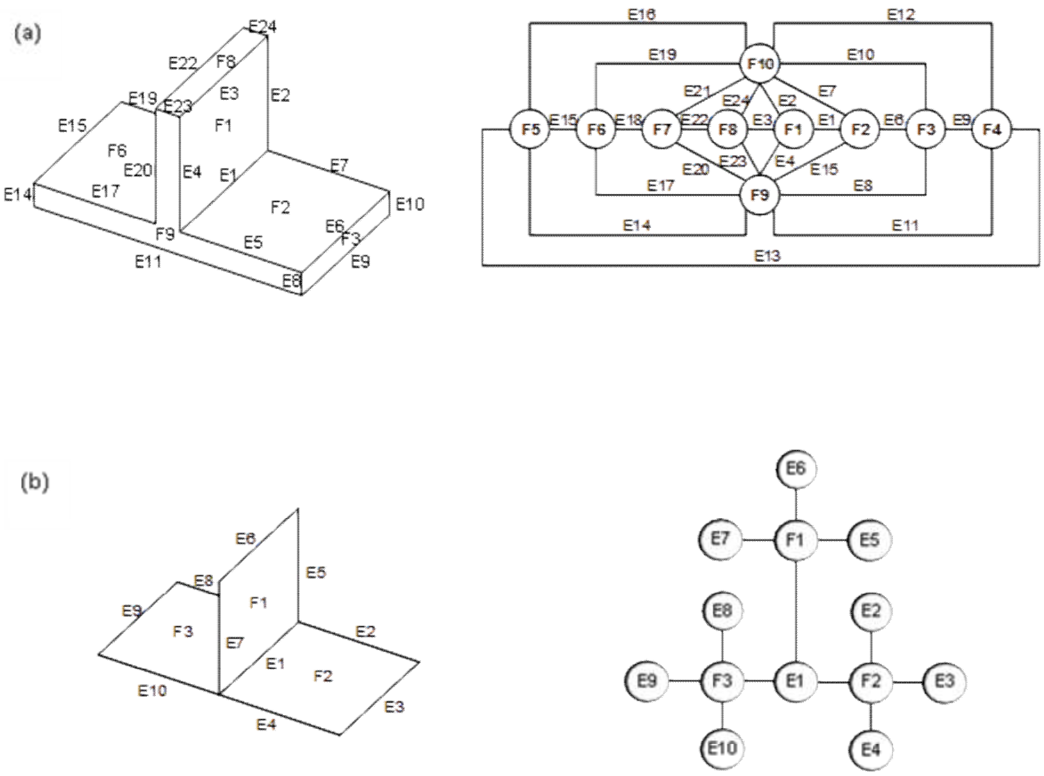
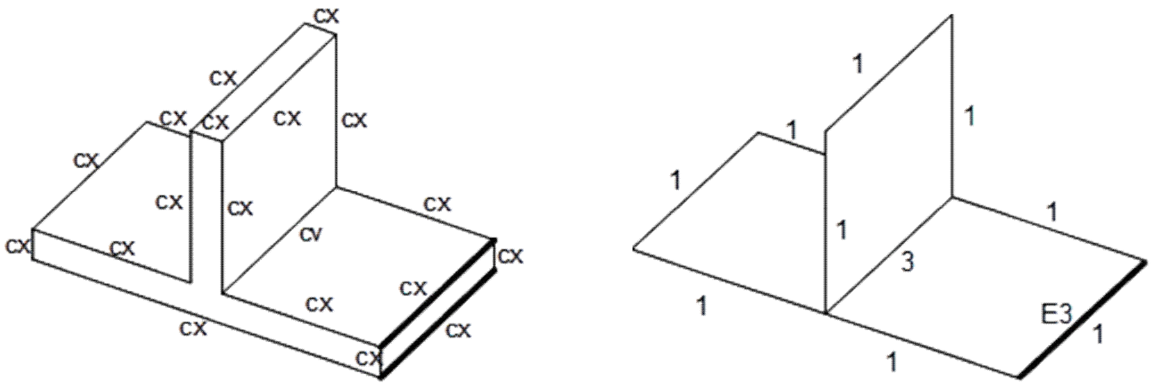


Figure 4. Example Solid and Mid-Surface Models with Geometry Graphs

3.3.2 Overview of Topological Similarity Evaluation Method

The topological similarity methodology proposed in this paper uses edge grouping to allow the graph attributes of the two models to be compared. Figure 5 (a) shows the solid model of the T-shaped part with attributes showing the concavity (cv) or convexity (cx) of each edge and Figure 5 (b) shows the mid-surface model of the part with attributes showing the order of each edge.



(a) **(b)**

Figure 5. (a) Solid model of T-junction part showing concave/ convex attribute for each edge (b) mid-surface model showing the order of each edge.

It can be seen from the figure that each mid-surface edge is in close proximity to a group of 2 or more solid model edges. The topological similarity between the mid-surface model and the solid model can be evaluated by comparing the convexity/ concavity characteristics of each group of solid edges to the order of each corresponding mid-surface edge. For example in Figure 5 the mid-surface edge E3 has order 1 and corresponds to a group of 2 convex solid model edges in close proximity to it (highlighted in bold). A solid model edge is said to be in close proximity to a mid-surface edge if the distance between the two edges is less than the local wall thickness along the entire length of the mid-surface edge.

For thin walled parts the relationship between mid-surface and solid model edges can be defined as follows. The order of a mid-surface edge is equal to the number of solid edges in close proximity to it plus an angle factor. The angle factor is required to account for junctions where the faces form an angle of 180°. The relationship is valid for all edge junctions with one or more concave edge. Formally the relationship can be stated as:

For G_i where $(N_{cv} > 0)$

$$O_i = N + A \quad \textbf{(4)}$$

For G_i where ($N_{cv} = 0$)

$$O_i = 1$$

Where

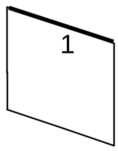
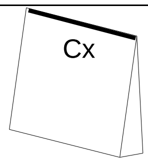
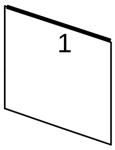
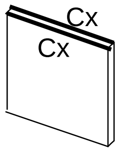
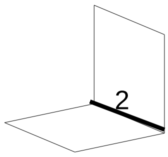
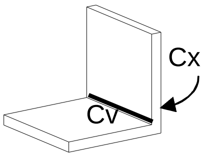
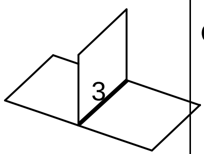
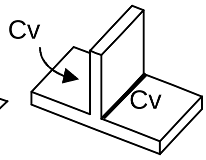
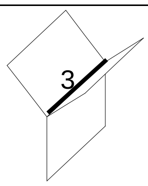
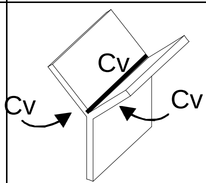
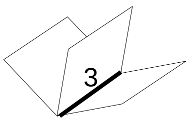
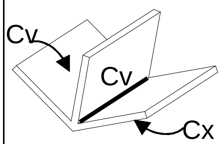
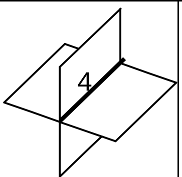
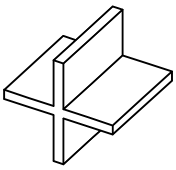
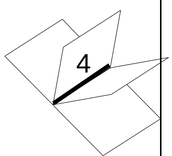
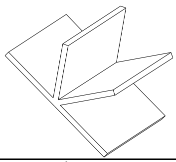
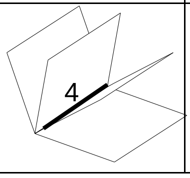
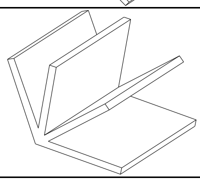
O_i = Order of mid-surface edge *i*
G_i = Group of solid edges in close proximity to mid-surface edge *i*
N = Number of solid edges in group G_i
N_{cv} = Number of concave solid edges in group G_i
N_{cx} = Number of convex solid edges in group G_i
A = Angle factor calculated from the sum of concave solid edge angles in G_i:

$$A = \begin{cases} 0 \quad if \quad \sum_{j=1}^{N_{cv}} \alpha_j \neq 180° \\ 1 \quad if \quad \sum_{j=0}^{N_{cv}} \alpha_j = 180° \end{cases}$$

α_j = face angle for concave solid edge *j*

Table 1 illustrates the relationships for some common example junction types found on thin walled parts.

Table 1. Solid and Mid-surface Model Edge Attributes.

Connection type	Mid-surface Model showing shared edge order *(O_i)*	Solid model	Number of Solid Edges in group G_i (N)	Number of Concave Edges in group G_i (N_{cv})	Sum of Concave Edge Angles ($\sum \alpha$)	Angle Factor *(A)*	Factored Solid Edges *(N + A)*
Unconnected Sharp Edge			1	0	0°	N/A	N/A
Unconnected Edge			>=2	0	0°	N/A	N/A
L-Junction			2	1	90°	0	2
3-way (T)			2	2	180°	1	3
3-way (Y)			3	3	360°	0	3
3-way (acute)			3	2	135°	0	3
4-way (X)			4	4	360°	0	4
4-way (fan)			3	3	180°	1	4
4-way (acute)			4	3	135°	0	4

3.3.3 Topological Similarity Index (TSI)

A global measure of topological similarity can be defined as the proportion of solid edge groups G_i with correctly matched topology to the mid-surface model.

$$TSI = \frac{|G_{pass}|}{I} \qquad \textbf{(5)}$$

Where

$|G_{pass}|$ = Number of edge groups G_i with correct topology (calculated using (4)).

I = Total number of mid-surface edges i in model

The TSI provides a global measure of topological similarity between a solid and mid-surface model. A TSI of 1 indicates a high degree of topological similarity between two models, and a TSI of 0 indicates that there are no topologically matched edges between the models.

3.3.4 Combined Measure of Similarity

The geometric and topological similarity measures can be combined to provide an overall similarity index (OSI). The overall similarity is defined as the product of the geometric (GSI) and topological (TSI) indices to capture the dependence of the topological similarity on the geometric similarity:

$$\text{OSI} = \text{GSI x TSI} \qquad \textbf{(6)}$$

3.3.5 Further Considerations for Similarity Evaluation of Practical Designs

3.3.5.1 Parts with Variations in Wall Thickness

The proposed geometric similarity evaluation method provides a good measure of solid/ mid-surface similarity for thin walled parts with small variations in wall thickness. However, a potential limitation to the method is that is uses a single threshold value to compare the models, which means that the results will be less accurate for parts with significant variations in wall thickness. The difficulty is that the chosen threshold value must be large enough to accommodate the maximum wall thickness in the model, but using a large threshold value will mean that small differences cannot be identified on thinner regions of the model.

The methodology could be extended to allow more accurate evaluation of parts with variations in wall thickness by introducing local threshold values. In this approach the model would need to be partitioned into regions based on the local wall thickness prior to performing the similarity evaluation. The similarity evaluation could then be performed on a region by region basis using local threshold values. Further work would be required to investigate how best to partition the models into appropriate regions for evaluation. Local threshold values have not currently been implemented in the demonstrator.

3.3.6 Parts with Filleted Edges

The topological similarity method utilises patterns of convex and concave edges at wall junctions to perform the similarity evaluation. The current methodology cannot be used directly on parts with filleted edges because on these parts the concave and convex edges are replaced by pairs of tangent edges. The methodology could be extended to support filleted edges by checking for concave and convex fillet faces between pairs of tangent edges, however, care would need to be taken because the introduction of fillets also increases the required search radius (and hence threshold value) when searching for close edges. The evaluation of filleted edges has not currently been implemented in the demonstrator.

4 Implementation of Demonstrator

4.1 Geometric Similarity

A demonstrator for the geometric similarity evaluation has been implemented using C++ and the CAD system UGS I-DEAS NX. A flow chart of the evaluation process is shown in Figure 6.

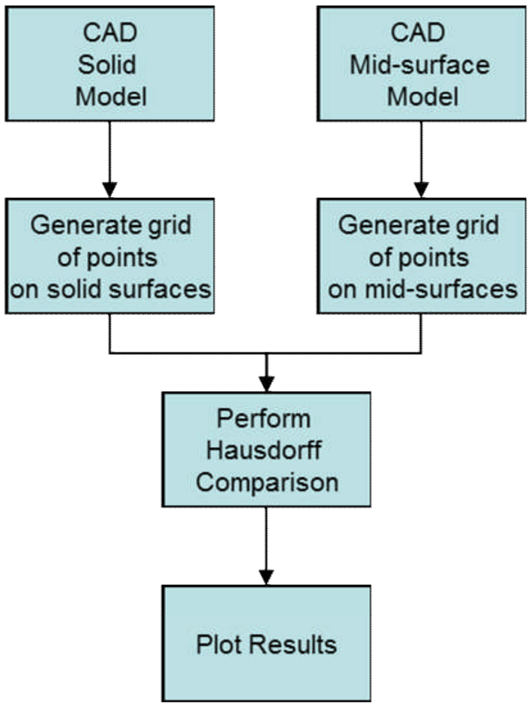


Figure 6. Flow Chart of Geometric Similarity Evaluation

For the demonstrator the grids of points on the mid-surface and solid models and have been generated using the automatic finite element mesh generation function in UGS I-DEAS, however a wide range of other techniques could be used. The Hausdorff comparison has been performed using a simple algorithm similar to that presented by Gregoire and Bouillot [28]. The Hausdorff algorithm is shown below:

```
Algorithm Find_Minimum_Distances
    begin
        Let X be an array of all the solid model points
        Let Y be an array of all the mid-surface model points
        Let MAX-DIST be an array of the maximum distances
        for each point x in X do
            MAX-DIST(x) = 0
```

```
                for each point  y in Y do
                        TEMP-DIST = distance(x,y)
                        if TEMP-DIST < MAX-DIST(x) then
                                MAX-DIST(x) = TEMP-DIST
                        end
                end
        end
end
```

The minimum distance values are stored in an array and can be plotted to give a graphical representation of the distribution of minimum distances between the two models. The point density for the Hausdorff comparison must be carefully selected to ensure that it is fine enough to resolve differences between the models without excessive computation time. For testing the point density has been defined to be equal to 1.1 x the wall thickness.

At present only the solid to mid-surface comparison has been implemented in the demonstrator because from experience the most common form of mid-surface error is for the mid-surface model to be generated with missing surfaced, however the reverse evaluation could easily be added to the implementation.

4.2 Topological Similarity

The topological similarity evaluation has been implemented using C++ with CAD integration via STEP. The program performs the topological similarity evaluation using graph attributes of the solid and mid-surface models as described in section 3.2.1. The code has been integrated with the feature recognition software previously developed by the author [29] which provides an architecture to store the models in an appropriate form for evaluation. The similarity evaluation results are output as a STEP file and a printed report. The STEP results file is colour coded to show correctly connected edges (green), incorrectly connected edges (red) and unevaluated edges (yellow), silhouette edges and surface iso-curves (orange). The unevaluated edges are edges which are shorter than the threshold value and closed circular edges which are currently not supported by the demonstrator. Unevaluated edges are disregarded during the evaluation.

The procedure for evaluating the topological similarity of a solid and mid-surface model has been implemented as follows:

1. Construct an Attributed Face Adjacency Graph for the solid model and store convexity/ concavity attributes. The convexity/ concavity attributed are evaluated using a method based on [30].
2. Construct an Attributed Mid-Surface Adjacency Graph for the mid-surface model and store edge order attributes. The edge orders are evaluated using the method previously presented by the author in [5].
3. Process the solid and mid-surface model graphs to create a group of solid model edges in close proximity to each mid-surface model edge.
4. Compare the order of each mid-surface edge to the convexity/ concavity characteristics of the solid edges in its associated solid edge group.

The topological similarity evolution is shown in the flow chart shown in figure 7:

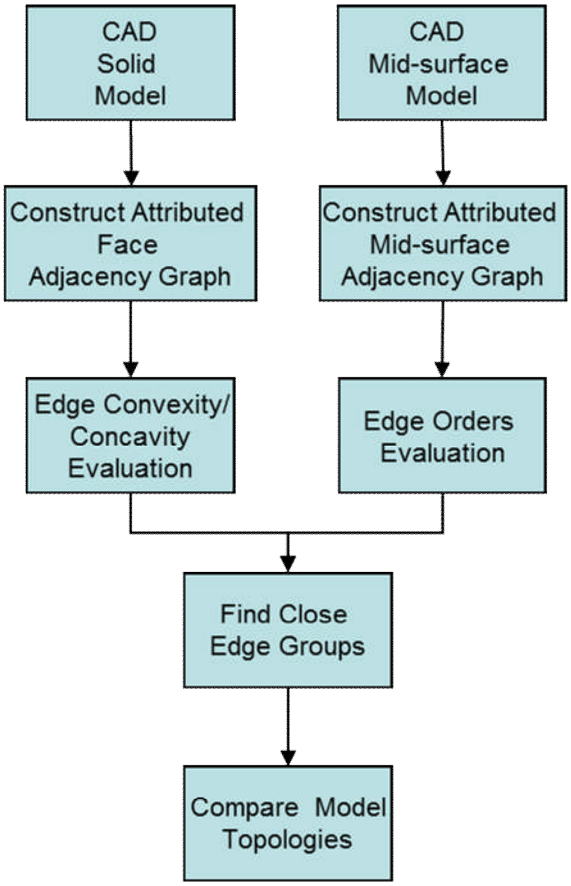


Figure 7. Flow Chart of Topology Evaluation

At present the demonstrator is able to evaluate parts with planar faces but it could be extended to support curved geometries.

The Find_Close_Edges algorithm identifies the group of solid edges that are in close proximity to each mid-surface edge. The algorithm performs the identification in two parts – firstly identifying candidate edges which are close to the start vertex of each mid-surface edge, then testing the candidate edges to ensure that they are close to the mid-surface edge at both ends. The algorithm is presented below:

Algorithm Find_Close_Edges
begin
(1) Let MID-EDGES be an array of all the mid-surface edges
(2) Let SOLID-EDGES be an array of all the solid edges
(3) Let WALL –TK be the maximum wall thickness for the part
(4) **for** each edge *i* in MID- EDGES **do**
(5) CAND-EDGES[*i*] = {}*; array of candidate edges*

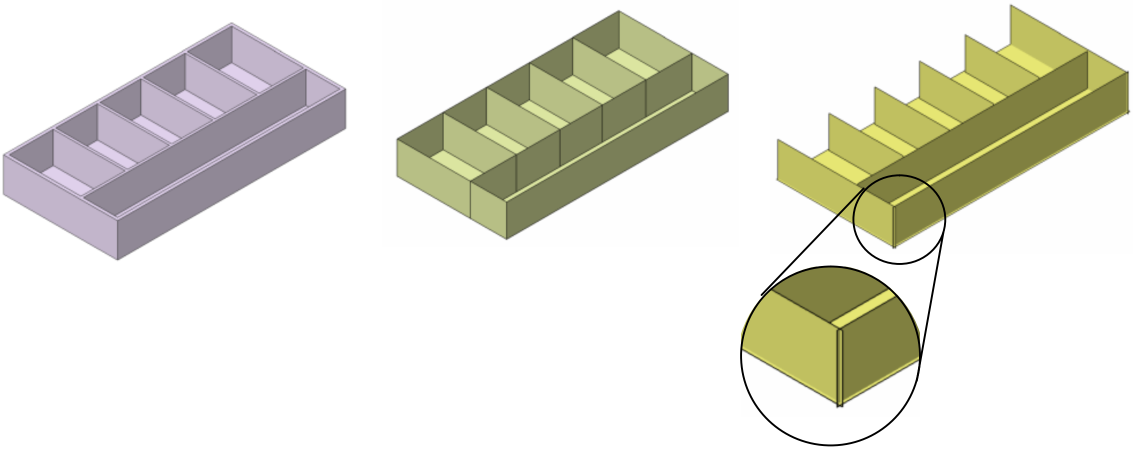
```
                CLOSE-EDGES[i] = {}; array of close edges
                Let START-ME be the start vertex for i
                for each edge  j in SOLID-EDGES do
                        Let START-SE be the start vertex for j
                        Let END-SE be the end vertex for j
                        START-DIST= min_distance (START-ME, j)
                        if START-DIST <= WALL-TK then
                                Append j to CAND-EDGES[i]
                        end
                end
                Let END-ME be the end vertex for i
                for each edge k in CAND=EDGES[i] do
                        END-DIST = min_distance(END-ME, j)
                        if END-DIST <= WALL-TK then
                                Append k to CLOSE-EDGES[i]
                        end
                end
        end
end
```

5 Test Cases

The mid-surface quality evaluation techniques have been tested on a range of thin walled parts. The analysis for one part is presented in detail in section 5.1.1, and the results for six other parts are included in section 5.1.2.

5.1.1 Test Case 1

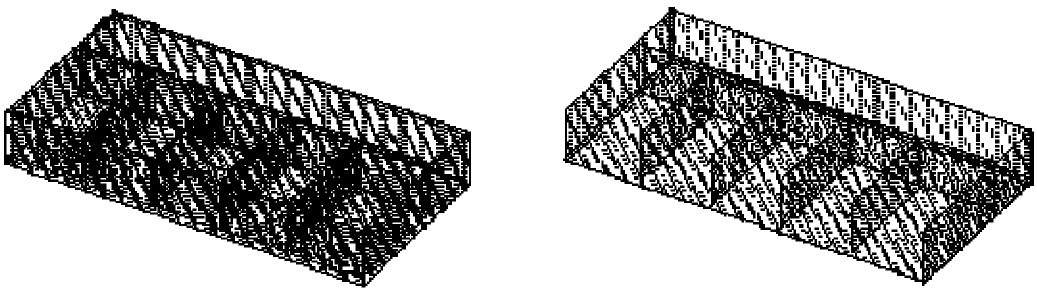
The first test case is a simple box with 6 internal compartments. In order to demonstrate the similarity measures the evaluation has been performed using two versions of the model - firstly an accurately generated mid-surface abstraction (test case 1A), and secondly a mid-surface model with some typical model errors (a missing face and incorrect edge connectivity) (test case 1B). The part has an external wall thickness of 2.5 mm and internal wall thickness of 2.0 mm. The solid model and generated mid-surface models are shown Figure 8.



(a) **(b)** **(c)**

Figure 8. Test Case 1. (a) Solid Model, (b) Correctly Generated Mid-surface Model (1A), (c) Incorrectly Generated Mid-surface Model (1B)

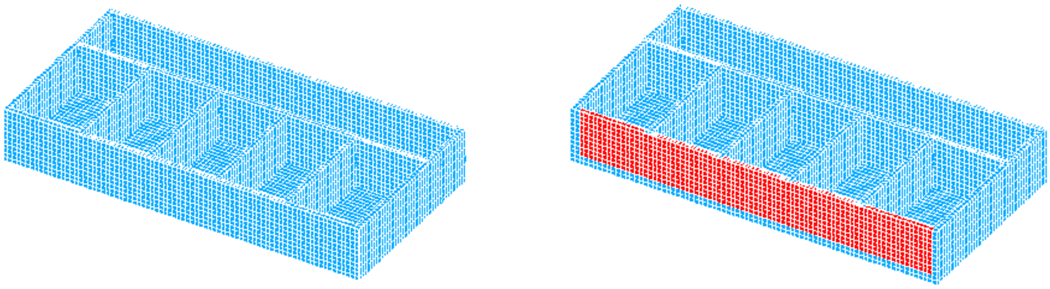
The grids of points generated on the model surfaces for test case 1B are shown in Figure 9. The points were generated with a spacing of 2.5 mm (equal to the wall thickness). The solid model has 15927 points and the mid-surface models 8168 and 7636 points respectively.



(a) **(b)**

Figure 9. Points used for Hausdorff comparison for Case Study 1B. (a) Solid Model Points (b) Mid-surface Points

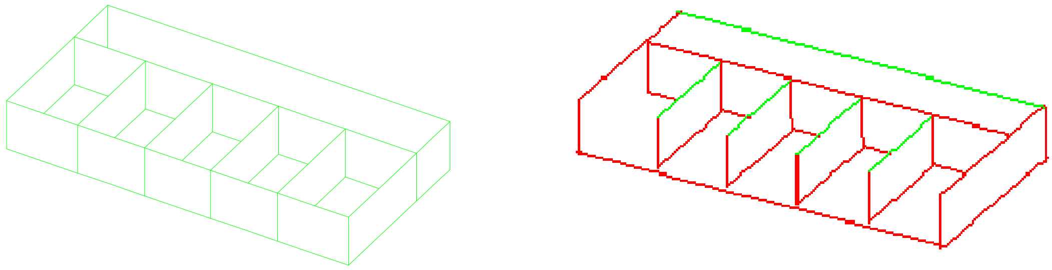
The geometric similarity evaluation has been performed for the two versions of the mid-surface model and the results are shown in Figure 10. The results show that model 1A has excellent geometric similarity to the solid model, with a maximum distance from any point on the solid model to the mid-surface model of 2.16 mm. Model 1B has good similarity to the solid model except in the region of the missing face where the maximum distance from the solid model to the mid-surface model is 19.3 mm. Notice that the geometric similarity measure is not sensitive enough to identify the small connectivity errors between adjacent edges in the model (as shown in Figure 8 (c)). The distance results have been plotted on the solid model geometry with a threshold value of 2.5 mm to allow the location of the geometric dissimilarity to be visualised. The GSI for test case 1A is calculated to be 1 and for test case 1B is 0.9.



(a) (b)

Figure 10. Visualisation of Geometric Similarity Results for Test Case 1. (a) Test Case 1A (b) Test Case 1B

The topological shape evaluation has also been performed for the two models. Figure 11 shows the topological similarity results for both models. From the figure it can be seen that test case 1A all edges match the expected topology (indicated by the colour green), whereas for test case 1B only 5 edges are correctly connected (incorrectly connected edges shown in red). The TSI for test case 1A is computed to be 1, whereas the TSI for test case 1B is 0.14. The low TSI value for test case 1B is caused by the poor connectivity between the faces of the mid-surface model.



(b) (b)

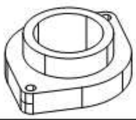
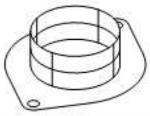
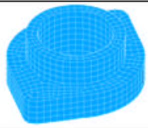
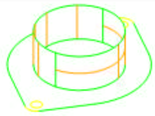
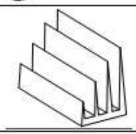
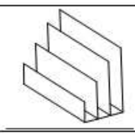
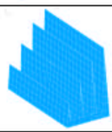
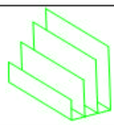
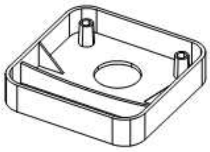
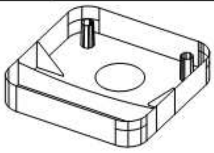
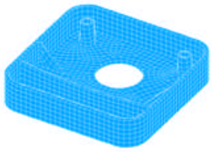
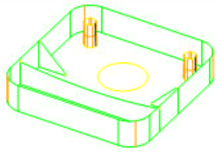
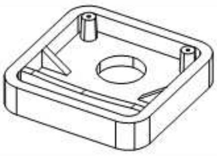
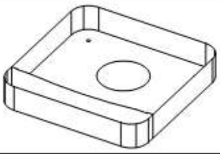
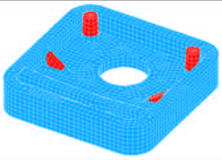
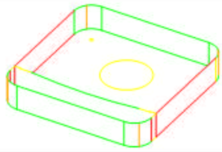
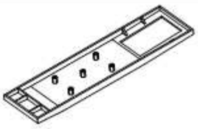
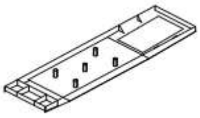
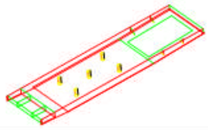
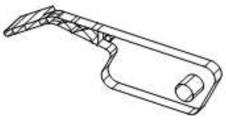
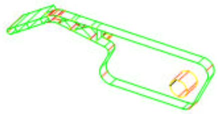
Figure 11 Topological Similarity Results for Test Case 1. (a) Test Case 1A (b) Test Case 1B.

The results for this test case illustrate the importance of evaluating both topological and geometric similarity. For test case 1B the geometric similarity is relatively good, but the topological similarity is very poor. The overall similarity for the two models can be calculated to be 1 for test case 1A and 0.13 for test case 1B.

5.1.2 Further Test Cases

The results for six other test cases are shown in Table 2. The test cases have been selected to demonstrate a range of mid-surface geometries and some representative errors in mid-surface generation including geometric errors (missing regions), topological errors (incorrect connectivity) and combinations of both types of errors.

Table 2 Test Case Results

Test Case	Solid Model	Mid-Surface Model	Geometric Similarity Results	Topological Similarity Results	Similarity Indices
2					GSI = 1 TSI = 1 OSI = 1
3					GSI = 1 TSI = 1 OSI= 1
4					GSI = 1 TSI = 0.95 OSI = 0.95
5					GSI = 0.96 TSI = 0.73 OSI = 0.7
6					GSI = 0.95 TSI = 0.41 OSI = 0.4
7					GSI = 1 TSI = 0.86 OSI = 0.86

The table shows the geometric and topological similarity for each test case. Missing regions of the mid-surface model and incorrectly connected mid-surface edges have been correctly identified, and an overall measure of similarity is provided for each part. The demonstrator is able to evaluate parts with planar and curved faces, although the topological similarity evaluation does not currently support closed circular edges (shown in yellow in test cases 4 – 7). However if the circular edges are split into pairs of semi-circular edges (as shown in test case 2) the edges are correctly evaluated. Test case 3 provides an example of a component with four sharp edges and variations wall thickness.

6 Summary and Conclusions

This paper has described two similarity measures for comparing mid-surface and solid models. The literature review highlighted that geometric similarity is not sufficient for evaluating mid-surface model quality, and that in order to obtain useful results it is necessary to use a topological measure of similarity in combination with the geometric similarity evaluation.

The geometric similarity measure proposed in this research is based on the Hausdorff distance and requires the models to be discretised into grids of points lying on the model surfaces. The topological similarity evaluation technique uses geometry graph attributes to compare the edge topologies of the two models. The use of both techniques together gives significant advantages over performing only a geometric similarity evaluation because small geometric differences that may not be identified by a geometric method can be identified as differences in model topology.

The GSI and TSI provide a global measure of model similarity and can give confidence to an analyst that a mid-surface model is representative of its parent solid model. The graphical display of local dissimilarity helps the analyst to identify the location of any errors on the mid-surface so that the model can be modified to be more representative if required. The similarity measures have been found to be effective for identifying errors in mid-surface models and are applicable to a range of practical designs.

The geometric similarity evaluation is generic and applicable to any thin walled part. The main limitations are the difficulty in evaluating parts with widely varying wall thickness and in identifying features that are small in comparison to the local wall thickness. The topological similarity evaluation has some limitations in its current form but has been demonstrated to be applicable to a range of practical parts. In particular the current implementation of curved edge evaluation is simplistic (using only the end and mid-points for comparison) and support for filleted edges has not yet been implemented. One other potential limitation of the presented approaches is the computational cost of performing the evaluation for complicated parts; however in testing to date the time taken to perform the evaluations has been found to be acceptable, with all comparisons to date performed in less than two minutes on a Pentium P4 PC.

6.1 Future Work

The methodologies presented in this paper provide an initial proposal for mid-surface model similarity evaluation, but further work would be required to develop a fully functional method, in particular:

- A more sophisticated implementation of curved part checking to fully evaluate the curve-curve distances
- Further investigation into the evaluation edge fillets
- Investigation into the use of local threshold values to provide better support parts with variations in wall thickness.

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